# Barlow's Formula or Mariotte's Formula?

It is well known that the calculation of the hoop stress and of the thickness in piping subjected to internal pressure is based on the following formula (where: $P$ is the internal pressure, $D_i$ is the inside diameter, $t$ is the thickness):

$$\sigma_h = \frac{PD_i}{2t}$$

The history of this formula, despite of its simplicity, even on the conceptual standpoint, is however unexpectedly jagged and sometimes controversial.

The subject, indeed, even though apparently consolidated by, I would say, a secular use, is really affected by some confusion due to the various interpretations that over the years have superimposed each other and stratified (on this subject see ref. [4]).

A denomination issue is, first, faced by whom have studied engineering in Italian Universities. In Italy, the designation "**Mariotte's formula**" is assigned to the equation for computing the primary hoop stress in cylindrical shells, whereas, in English technical literature, the designation "**Barlow's formula**" is assigned to the same equation. Outside Italy, the only known "Mariotte's formula" is the one for compressed gases. Based on what I know, the same occurs even in France.

Going back to the origins, that is to textbook "*Traité du mouvement des eaux*" by Mariotte (1700), ref. [1] (>>), at page 348 we find the following paragraph: *« II. Discours, De la force des Tuyaux de conduite, et de l'épaisseur qu'ils doivent avoir suivant leur matière et la hauteur des réservoirs »*, whose reading does not provide any formula, but only the following rule: « *I. Règle : Si la hauteur du réservoir est double, il y aura deux fois autant de poids d'eau, & par conséquence il faudra deux fois autant d'épaisseur de métal dans le tuyau afin qu'il y ait deux fois autant de parties à séparer. Si le diamètre du tuyau est 2 fois plus large, il faudra 2 fois plus d'épaisseur : car les mêmes parties du fer blanc ne feront pas plus chargées, & elles sont seulement doubles.* ». This rule exhibits *in nuce* some of the elements that will be found in the formula, but not all of them, since, for example, the principle (equilibrium) is not declared, and the material strength is missing.

On the other side, Barlow in his textbook "*A Treatise on the Strength of Timber, Cast Iron, Malleable Iron and Other Materials, with rules for application in architecture, the construction of suspension bridges, railways, etc.; and an appendix on the power of locomotive engines, and the effect of inclined planes and gradients*", 1837, ref. [2] (>>), at page 205, in paragraph "*On the Strength of Hydrostatic Presses*", obtains the following formula (see page 210 of the treatise), which is very close to the one that today is designated as "Barlow's formula" but not identical to it:

$$x = \frac{pr}{c - p}$$

where: $r$ is the internal radius, $c$ is the material cohesive strength, $x$ is the thickness and $p$ is the internal pressure.

The current "Barlow's formula" has been obtained by **Goodman** who, in his textbook "*Mechanics applied to Engineering – 1914*", ref. [5], upon application of the so called Barlow's theory to thick-walled piping, gets the formula $pr_o = f_i t$ where we find the outer radius, $r_o$, and the hoop stress, $f_i$, on the internal edge.

Goodman obtained this equation by imposing that the hoop stress on the internal edge of a thick-walled piping be equal to the hoop stress of a thin-walled piping, where really that computed is the stress averaged over the thickness, not a local stress. For the thin-walled piping, even Goodman, based on equilibrium considerations, obtained the same equation shown at the beginning of this article, with the inside diameter or radius, that implies that the computed stress is averaged over the thickness. A consequence of the Goodman's equivalence is that if the hoop stress on the inside edge is shown, then in the formula the outer diameter shall appear, since the theory used to derive it is based on the principle that the hoop stresses have values in inverse proportion to the square of the radius where it is computed (the inside stress stands to the outside stress alike the square of the outer radius stands to the square of the inner radius). Should Goodman have imposed the equivalence with the thin-walled piping considering the average stress of the thick-walled piping, he would have obtained the equation with the inside diameter for the thick-walled cylinder, too. All these considerations are developed in detail in the technical paper (About Barlow's and Mariotte's Formulas) attached to this article (downloadable by whom is interested).

In technical literature (see attached paper) three "Barlow's formulas" are considered: with the inside diameter, with the mean diameter and with the outside diameter (which is the most frequent one). The Barlow's formula is usually considered valid for thin-walled piping only ($D/t > 20$). This statement anyhow is strictly related on the diameter adopted and how the stress so obtained is classified and used, as a matter of fact:

1) **If the inside diameter is used, the equation gives the primary membrane hoop stress**, that is the hoop stress averaged over the thickness that equilibrates the end pressure force. This stress is correct whatever is the thickness since it depends only upon the equilibrium. It is the stress limited by the pressure vessels codes to control the plastic collapse:

$$\sigma_{h,m} = \frac{PD_i}{2t}$$

2) **If the mean diameter is used and the equation is furtherly solved in terms of the inside diameter, we get the Boardman's formula** (rif. [3]), substantially used by all design codes, with $Y = 0.6$ for ASME VIII-1 and $Y = 0.5$ for EN 13445-3:

$$S = \frac{pR_i}{t} + Yp$$

It is noted that, if we set $Y = 1$, we get the original Barlow's formula. The **Boardman's formula** may be derived by the membranal theory (rif. [6]), therefore it applies only to thin-walled piping. The factor $Y$ however may be so calibrated to extend the validity also in the field of non-thin walled piping, as ASME VIII-1 does setting $Y = 0.6$ and extending the formula validity up to $t/R \leq 0.5$.

3) **If the outside diameter is used, the formula obtained by Goodman is got**:

$$\sigma_{h,i} = \frac{PD_o}{2t}$$

This formula provides the maximum hoop stress on the inside edge, whose value is increasingly lesser than the correct one as the thickness increases. It follows that this version of the Barlow's formula rigorously applies only to thin-walled piping.

Even though the same hoop stress formula is used for either cylindrical pressure vessels or piping, the designation "Barlow's formula" is almost exclusively adopted for piping only in both technical literature and standards / codes; practically, no pressure vessel design handbook and related standard / codes makes use of this designation.

The Barlow's formula is used also to evaluate the **piping limit pressure (*bursting pressure*)**.

Two limit states are to be considered for piping: the elastic service limit state (gross deformation or plastic collapse) and the collapse (bursting) limit state. The elastic service limit state is governed by the yielding strength; it is important in presence of threading connections (for example, the so called OCTG = *Oil Country Tubular Goods*) where the gross deformation may cause leakage. The collapse limit state is governed by a strength value that can be close to the ultimate tensile strength in case of materials having low strain hardening exponent, whereas has a value between yielding and ultimate tensile for other materials.

The search for the most suitable equation for the purpose to the determine the bursting pressure has been the subject of several studies over the years. Among the most recent ones, the work of Zhu and Leis (ref. [7]) has shown that the best estimate is obtained using the Barlow's formula with the ultimate tensile strength and the mean diameter:

$$P_b = \frac{2\sigma_u t}{D_m}$$

but only for materials having ratio $Y/T = 0.7 \div 0.9$ which is characteristics of carbon steels, being:

$$\frac{Y}{T} = \left(\frac{\varepsilon_{ys} e}{n}\right)^n$$

For steels having high strain hardening exponent, $n$, as the austenitic stainless steels, it is necessary to use more suitable equations that appropriately consider this exponent (Zhu-Leis, Klever, Faupel, etc.). In the last equation, $\varepsilon_{ys}$ is the yielding strain. For carbon steels, then we obtain: $\varepsilon_{ys} = 0.2\%$, $n \leq 0.1$, $Y/T \geq 0.745$.

The determination of the collapse (bursting) pressure is, however, a so sensitive and complex subject that cannot be run out with the few hints here above, especially for materials with high strain hardening exponent.

## Conclusions

1) It is not clear why in Italy the use of designating as Mariotte's formula what abroad is indicated as Barlow's formula was established.
2) The reference Barlow's formula should make use of the inside diameter which returns the primary membrane hoop stress, due to the equilibrium, and which is valid whatever is the thickness.
3) The Barlow's formula currently used for piping makes use of the outer diameter and is, therefore, applicable to thin-walled piping only, since it provides the maximum hoop stress on the inner edge whose value is strongly dependent on thickness.
4) The Barlow's formula with the mean diameter leads to the Boardman's formula which is basic for the rules of all pressure vessels and piping design codes.
5) The Barlow's formula with the ultimate tensile strength and the mean diameter gives reliable values for the collapse (bursting) pressure of piping made with steels having strain hardening exponent $n \leq 0.1$ (carbon steels); on the contrary, it cannot be applied to steels having high strain hardening exponent, as austenitic stainless steels.

## References


[1] Edme Mariotte - Traité du mouvement des eaux – Jean Jombert, 1700

[2] Peter Barlow – A Treatise on the Strength of Timber, Cast Iron, malleable Iron and Other Materials, with rules for application in architecture, the construction of suspension bridges, railways, etc.; and an appendix on the power of locomotive engines, and the effect of inclined planes and gradients. – London, John Weale, 1837

[3] Boardman, H. C., "Stresses at Junction of Cone and Cylinder in Tanks with Cone Bottoms or Ends", Water Tower, 1944 (Also published in ASME, Pressure Vessel and Piping Design, Collected Papers, 1927-1959).

[4] A.J. Adams, K.C. Grundy, and C.M. Kelly, Nexen petroleum U.K. Ltd.; B. Lin and P.W. Moore, United States Steel Corporation – The Barlow Equation for Tubular Burst: A muddled History – IADC/SPE-189681-MS, 2018

[5] Goodman, John – Mechanics applied to Engineering – 1914

[6] Bednar, H.H. – Pressure Vessel Design Handbook – Second edition – Krieger Publishing, 1991.

[7] Xian-Kui Zhu, Brian N. Leis - Evaluation of burst pressure prediction models for line pipes – Elsevier, International Journal of Pressure Vessels and Piping xxx (2011) 1-13. [89: 85-97, 2012]


## Attachment

Technical paper – About Barlow's and Mariotte's Formulas