About Barlow’s and Mariotte’s Formulas

Introduction

In the technical literature there is a subject that, despite its apparent universality and simplicity, is sometimes given opposite interpretation and application. It is the classical formula used to calculate the hoop primary membrane stress in piping or cylindrical vessel loaded with an internal pressure:

$$\sigma_h = \frac{PD_i}{2t}$$

This formula represents the basis for the design (and verification) criteria adopted by practically all construction and design codes for pressure vessels and pressure piping.

A first peculiar aspect is about the name this formula is given to, since in the international English technical literature it is, with no exception, given the name of “**Barlow’s formula**”, whereas in Italian academy and handbooks written by Italian authors, even with abroad publishing companies and in English, it is designated as “**Mariotte’s formula**”. On this side, some author moves forward up to assigning to Boyle too the paternity of this formula, designating it as “**Boyle-Mariotte’s formula**” (see below). For what I know, in Italy nobody uses the wording “Barlow’s formula”. I too during my university degree became familiar with “Mariotte’s formula” so that I used this expression to name the hoop stress equation. Leaving out the designation “Boyle-Mariotte’s formula” whose genesis I can only attribute to a cognitive bias or a mental association lapsus, due to the fact that Boyle had no involvement in material strength studies (see Timoshenko ref. [3]), especially for piping, what needs to be understood is the reason of this different designation of the same mathematical object and what is the correct one.

A second peculiar aspect that deserves a deep review has a technical nature and involves, in addition to the parameters used in the equation (especially if the internal, external or mean diameter), whether it is applicable to thin or thick wall cylindrical shells.

Mariotte and Barlow

In the title of this paragraph the order of citation of the author’s follows the time they operated. Edme Mariotte, member of the French Academy, was active in France in the seventeenth century (1620-1684); whereas Peter Barlow was active in England at the turn of the eighteenth and nineteenth centuries (1776-1862).

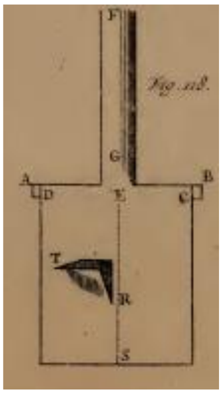


Figure 1 – Cylindrical vessel used by Mariotte for his bursting tests

The former is qualified as physicist (ref. [1]; whereas, the latter as mathematician (ref. [2]). In reality, in accordance with the habits of the time, both moved overoperated on various scientific fields ranging from physics, to engineering, to astronomy.

In his book “*History of Strength of Materials*” (ref. [3]), Stephen Timoshenko greatly emphasizes Mariotte, described as one of the major physicists who in the seventeenth century contributed to the development of the strength of material science. Paragraph 5 of Chapter I, from page 21 through 25, is entirely dedicated to Mariotte and the main outcomes of his activity included the one linking his name to piping strength: the assessment of piping bursting strength when submitted to an hydrostatic pressure (Figure 1). Performing these tests, Mariotte reached the conclusion that the required thickness for the piping shall be proportional to the internal pressure and to the piping diameter.

This experiment is described in the second tome of the collection of his books, (ref. [4]), fifth part with title “*De la Conduite des Eaux et de la Resistance des Tuyaux*”, where at page 473 the following rule is read:

« *I. Règle : Si la hauteur du réservoir est double, il y aura deux fois autant de poids d’eau, & par conséquence il faudra deux fois autant d’épaisseur de métal dans le tuyau afin qu’il y ait deux fois autant de parties à séparer. Si le diamètre du tuyau est 2 fois plus large, il faudra 2 fois plus d’épaisseur : car les mêmes parties du fer blanc ne feront pas plus chargées, & elles sont seulement doubles.* »

Mariotte’s conclusions are then conceptual and not channeled into a formula.

Barlow too is mentioned in Timoshenko's handbook in relation to his treatise on material strength (ref. [6]), the work (calculations and tests) performed with Thomas Telford on the Menai Strait bridge (the first big suspension bridge), his contributions on the bending of beams. Beyond the strength of materials, Barlow worked on mathematics (*Barlow's tables*), optics (he invented the *Barlow's lens*), magnetism (conceived the Barlow's magnetic compensator), electro-magnetism and railway engineering. He also invented the so called "*Barlow's wheel*", a very initial application of a homopolar electrical motor (ref. [7], which Faraday used for his studies on the electro-magnetism.

At page 210 of his treatise (ref. [6], whose first edition was of 1837, the formula to compute the thickness of a cylindrical subject to hydraulic pressure is given. The formula is written as follows (using the same symbology as Barlow):

$$x = \frac{pr}{c - p}$$

where:

p is the pressure

r is the cylinder inside[1] radius

c is the material strength (designated as cohesion)

x is the searched thickness.

Barlow's conclusions are the same as Mariotte's ones, but more complete since related also to the material strength and presented with a mathematical formula. Formula that, however, is not only different than that today presented as Barlow's formula (se previous paragraph), but is also not correct since based on assumptions that do not respect the equilibrium, as Goodman proved in his book ref. [30] and the authors of paper ref. [22] support, along with standard ISO TR 10400:2018 (ref. [27]).

Citations of both formulas

As said at the beginning, the so called "Barlow's formula" is aimed at computing the hoop stress. In the Introduction, I assigned to this tress the specification "primary membrane" using a definition very classical in pressure vessel design. However, this description is often forgotten when speaking about this stress, what can be, in my opinion, one of the reasons of the confusion found in the use of the formula. It is, finally, interesting to observe that, while the hoop stress formula is present in all books and handbook, only those regarding piping design use the designation "Barlow's formula", whereas books and handbooks regarding pressure vessel construction and design usually do not use the wording "Barlow's formula" when speaking about the circumferential stress.

The equation for the calculation of the hoop stress (primary membrane) in a pressurized piping or cylindrical shell is given as "Barlow's formula" in the following documents (whose overview although extended is not to be intended as complete).

1. *The M.W. Kellogg Company - Design of Piping Systems* – Second Edition – John Wiley & Sons, 1956 ([9])

   Barlow mentioned in para. 2.1 page 32 (*outside diameter*), para. 2.4a page 43 where the following is read:

   "*For the most common surface of revolution, the cylinder, the so-called **inside diameter (or membrane)** and **outside diameter (or Barlow**) formulas were first used for thickness/diameter below and above 0.1, respectively. These were later supplanted by the mean diameter formula and, more recently, by the universally adopted formula approximating the results of Lamé formula. All these formulas may be expressed in a common manner as follows:*

   $$S = (pr_i/t) + Kp$$

   where:

   p = internal pressure

   r_i = inside radius

[1] Barlow clarifies that r is the inside dimeter at page 119 of ref. [31] that is an excerpt of the book ref. [6], in the paragraph preceding the one where the formula is given.

t = wall thickness

K = constant having values between 0 and 1

*If K is given the value of 0, the inside diameter formula is obtained; for $K = 0.5$, the mean diameter; far $K = 1.0$, the outside diameter. When the value of 0.6 is used, stresses are obtained which correlate reasonably well far values of t up to about $0.5r_i$ with **the recognized inside circumferential stress formula** of Lamé. This approximation discovered by H. C. Boardman (ref. [8]), was rapidly adopted far moderate temperature piping by both Pressure Vessel and Piping Codes, while far piping in the creep range it is considered applicable if a further adjustment of K is made as covered later in this section…"*

2. Peng, L.C., and Peng, T.L. – *Pipe Stress Engineering* – ASME Press, 2009 ([10])

Barlow mentioned at page 103, where the following is read:

"*Equation (4.7) **$t = PD/2SE$ (D = outside diameter)** is the simplified conservative formula generally referred to as **Barlow's formula**. This equation is the same as Eq. (2.14) $S_{hp} = r_i P/t$. Equation (4.7) can also be considered a special form of Eq. (4.5) $t = PD/2(SEW + Py)$ by considering the y coefficient as zero. Equation (4.7) is very conservative and is generally not used in creep range application. Due to its simplicity, this equation is used extensively in piping literatures.*"

3. Anvil – *Pipe Fitters Handbook, Building Connections That Last* – 06.05 ([11])

Barlow mentioned at page 208 (outside diameter) where the following is read:

"***Barlow's Formula** is a safe, easy method for finding the relationship between internal fluid pressure and stress in the pipe wall. The formula predicts bursting pressures that have been found to be safely within the actual test bursting pressures.*

*It is interesting to note that the formula uses the "outside diameter" of pipe and is sometimes referred to as the "**outside diameter formula**."*

$$P = 2tS/D$$

where:

P = *internal pressure*

D = *outside diameter*

t = *wall thickness*

S = *unit stress"*

4. Ellenberger, J. P. - *Piping and Pipeline Calculations Manual Construction, Design Fabrication and Examination* - Second Edition – BH, 2014 ([12])

Page 57, Figure 5.1, page 58, 59

*In calculating the wall thickness for pipe, the basic formulas for the **primary (hoop) stress** have been around for ages. There are many variations. At last count there were more than 20. Each of these addresses the basic problem somewhat differently to account for the variations in failure modes that can occur. But there are two fundamental differences: **the thin-wall approach, which we call the Barlow equation,** and **the thick-wall approach, which we call the Lame equation**. This then raises the question: When does a thin wall become thick? When the problem is thought about, it is not too hard to figure out that the pressure is higher on the inside of the pipe than on the outside. That may not be true if the pipe is buried in a very deep underwater trench. There, the outside pressure can be higher than the inside or at least the same order of magnitude.*

From that logic, for the more general case a man named Barlow surmised that if the pipe is thin one can assume that the thinness of that wall allows one to average the stress across the thickness (see Figure 5.1).

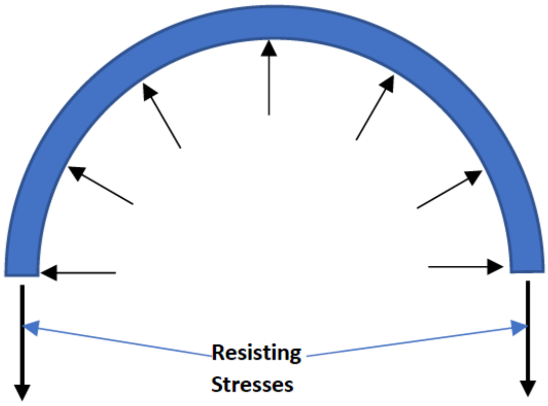


FIGURE 5.1 Barlow force diagram

So, he devised a simple formula by splitting a unit length of pipe through the diameter. He then said the pressure across that diameter creates a force equal to the pressure times the diameter, and the two-unit thicknesses create the area that resists that force.

*Thus, **the stress equation becomes: $S = PD/2t$**. This is the basic equation that the code presents. Since the goal is to find the unknown thickness, the formula is rearranged to solve for t given the other three parameters: pressure, outside diameter (OD), and allowable stress. The formula then becomes: $t = PD/2S$… for a given pressure the stress is proportional to the ratio: D/t*

Page 60,

"*A relationship between the thickness and the internal radius can be derived, and then this expression can be established:*

$$K = 1 + \frac{t}{r_i}$$

From this one can establish an index of the maximum stress to the internal stress and get an index of how much that maximum stress exceeds the simple Barlow equation (not the code-adjusted Barlow). Then, keeping in mind that the allowable stresses are established at a margin below yield, one can determine the severity of using the simpler equation"

Here the author introduces the Table 5.1 that provides how the ratio of Lamé maximum stress over Barlow's stress changes as a function of K. Barlow's stress is correctly indicated as an average stress, since the values shown in Table 5.1 are obtained making use of the inside radius, which is a way to obtain the average hoop stress (see below and Kellogg, page 43 at clause 1 here above)

Page 62 "*… like the Barlow equation, it is a good approximation …*"

Page 69 "*Use the simplest equation (Barlow) to calculate the thickness for a 6NPS pipe ($D_o = 6.625"$) at 875 psi …*"

Page 104 "…*The more recent finite-element programs, especially the solid-model ones with solid mesh, give those incremental stresses cell by cell. They do not make the assumption that Barlow did that it is okay to average. Nor do they make the Y factor adjustments that some codes make to set those stresses to some specific point through the wall. They require what is generally known as linearization to get from a comparable stress to a "code stress." …*"

Page 107 "*… 3. The next step is to calculate the pressure - that is, the "target pressure" or minimum proof test pressure. This calculation may be the most controversial, or the part of the test that is discussed the least. It is also the part that differs according to what type of fitting is being tested. The Barlow formula is used:*

$$P = \frac{2St}{D}$$

Page 111 "*…One can also calculate the pressure with the Barlow formula as follow …*"

5. Nayyar, M.L. – *Piping Handbook* – seventh edition – McGraw-Hill, 2000 ([13])

Barlow's equation at page C.22 (1251)

"***Pipe-Wall Thickness Selection.*** *After determining the internal diameter of the pipe, the designer must select materials, consider their strength, and select a pipe-wall thickness or schedule, as a function of temperature, pressure, corrosion, erosion, vibration, and external loads, as required.*

Pipe-wall thickness determination begins with the basic hoop stress in the pipe wall. This stress calculation ignores longitudinal wall stress that exists if the pipe has closed ends. An example of this is a flask or short header.

Advanced analysis shows that for thin-wall pipe, the outside diameter should be used in the hoop stress equation:

$$S = \frac{PD_o}{2t_{min}}$$

where

P = *internal design pressure, psig (kPa) [gauge]*

D_o = *outside diameter of pipe, in (mm)*

t_{min} = *minimum required pipe wall thickness, in (mm)*

S = *allowable stress, psi (kPa)*

This equation, called the ***Barlow formula****, is the basis for most code stress pipe-wall thickness calculations such as those provided in ASME B31.1 and B31.3.*

The formula also applies to thick-walled pipe.

The Barlow formula allows determination of wall thickness for flexible pipe required to handle internal pressure. …"

Note: The statement that using the outer diameter is a consequence of advanced analyses has no correspondence in any other technical book known. Using the outer diameter in this formula is a consequence of the equivalence adopted by Goodman (see the following paragraph Goodman 's demonstration) based on the stress on the inside surface rather than on the average stress. If the average hoop stress would be searched, the use of the outer diameter would cause a violation of the equilibrium for the thin-walled piping, too.

6. Helguero, M.V. - *Piping Stress Handbook* – Second Edition – Gulf Publishing Company, 1986 ([14])

Barlow's equation section 7 page 177:

"*The y value in the general formula reflects the effect of creep at high temperatures. In some ANSI Code sections that do not cover temperatures over 900'F, the y value of* 0.4 *is directly inserted in the formula, the formula with* $y = 0.4$ *is known as the "modified lame" formula. In Sections 4 and 8 of ANSI B31 the* ***Barlow formula is used****, which is a special case of the general formula in which* $y = 0.0$*. In addition, Section 8 is based on nominal thickness rather than minimum thickness; the tabulated* P/S *ratios for* $y = 0$ *may be placed on a nominal wall basis by multiplying by* $8/7$*.*"

7. Becht, C. IV – *Process Piping: The Complete Guide to ASME B 31.3* – ASME Press, 2002 ([15])

Chapter 4, page 26

"*… Three additional equations were formerly provided by the Code, but two were removed to be consistent with ASME B31.1 and simplify the Code. They may continue to be used. The first of the removed equations is*

$$t = \frac{PD}{2SE}$$

This equation is the simple Barlow equation, which is based on the outside diameter and is always conservative*. It may be used, because it is always more conservative than the Boardman equation, which is based on a smaller diameter (except when Y = 0). The second removed equation is*

$$t = \frac{D}{2}\left(1 - \sqrt{\frac{SE - P}{SE + P}}\right)$$

This equation is the Lamé equation rearranged to calculate thickness. Although it is not specifically included. it could be used, in accord with para. 300(c)3. However, it should not make a significant difference in the calculated wall thickness. …"

8. Rao, K.R. – *Companion Guide to the ASME Boiler and Pressure Vessel Code*, Vol. 2 – Fourth Edition – ASME Press, 2012 ([34])

Chapter 37, page 37.2 (717):

"*Design for internal pressure* (transportation pipelines)

*Hoop stresses due to internal pressure in pipelines are calculated using the "**Barlow equation**", $S_H = PD/2t$, and the outside diameter. The calculated hoop stress is an approximation to the exact hoop stress. Most pipelines have a ratio of diameter to wall thickness, D/t, in the range of 40 to 100 so the error in the approximation is small (1% to 3%) and is slightly conservative. Offshore pipelines, which use heavier-wall pipe, may be designed using the Lame equation.*

Page 37-5 (720):

Paragraph 403.2.1 (of B31.4) *establishes the design wall thickness of steel pipe as $t = P_i D/2S$ in accordance with the "**Barlow equation**" where the terms are as defined previously. The nominal wall thickness is then $t_n = t + A$, where A = sum of allowances for threading or grooving, corrosion, and increased thickness for mechanical protection against hazards. The pressure design equation applies to both straight pipe and curved pipe segments made by cold bending in the field or induction bending. The value of F used in B31.4 is 0.72 for all locations and fluids. …"*

Page 37-15 (730): (Offshore pipelines)

"*The hoop stress is calculated per A402.3.5 using the **Barlow equation**, but the net pressure is the difference between the internal operating pressure and the external hydrostatic pressure. The hoop stress design factor F_1 is 0.72 for the pipeline and 0.60 for the platform riser and piping. Design factors are listed in Table A402.3.5-1 and are reproduced herein as Table 37.5.*"

Page 37.18 (733): (B31.8)

37.3.2.2 Pressure Design Formula for Steel Pipe *The pressure design formula for steel pipe is specified in 841.1.1(a) in accordance with the "**Barlow formula**" as*

$$P = \frac{2St}{D} FET$$

where

P is the design pressure

S is the SMYS

t is the nominal thickness

D is the pipe specified outside diameter

F is the location class design factor obtained from Table 841.1.6-1

E is the longitudinal joint factor obtained from Table 841.1.7-1, and

T is the temperature derating factor obtained from Table 841.1.8-1.

The design pressure may or may not be the maximum allowable operating pressure (MAOP) of the pipeline as that is determined in consideration of the test pressure and the pressure ratings of components and equipment. …"

Page 37-27 (742): (B31.8)

"***37.3.4.1 Design of Plastic Pipe*** *The pressure design requirements for plastic pipe are found in Article 842.2. The formula for the design pressure is*

$$P = 2S\frac{t}{D-t} \times 0.32$$

where

S is a specified strength value (discussed below),

t is the specified wall thickness,

D is the specified outside diameter.

This equation is recognizable as the "Barlow equation" written for the mean diameter (D–t) rather than outside diameter*. For thermoplastic pipe, S is the long-term hydrostatic strength determined as a projection of short-term rupture strength tests to the intercept at 100,000 hours; and for thermosetting pipe it has a value of 11 ksi. The long-term hydrostatic strength is also known as the hydrostatic design basis (HDB). The 0.32 factor in the design equation corresponds to a nominal factor of safety of 3. For thermoplastic pipe, maintaining stresses at 32% of the HDB would ideally assure several hundred years of service owing to the inverse stress-time-to-rupture relationship of the viscoelastic material. The occurrence of leaks after a few years indicates elevated stress levels typically attributable to faulty installation. …*"

Page 37-31 (746):

"*The hoop stress is calculated per A842.2.2 using the* ***Barlow equation****, but the net pressure is the difference between the internal operating pressure and the external hydrostatic pressure. The hoop stress design factor F1 is 0.72 for the pipeline and 0.60 for the platform riser and piping. Design factors are listed in Table A842.2.2-1. Table A842.2.1 is identical to the analog offshore liquid pipelines, reproduced herein as Table 37.2. …*"

9. Rao, K.R. – *Continuing & Changing Priorities of ASME Boiler & Pressure Vessel Codes and Standards* – ASME Press, 2014 ([16])

   In the frame Chapter 11 "*Pipeline Integrity and Security*", paragraph 11.9 "*Defect Assessment Method*", subparagraph 11.9.6 "*Worked Example*", Page 11-33 (354), "*Example 3*".

   "*The hoop stress at the highest operating pressure is found from the* ***Barlow equation***

$$\sigma_p = pD/2t$$

   to be 37,440 psi and at the lower operating pressure = 4000 psi making the stress excursion per cycle equivalent to 33.44 ksi."

10. ASME B31.8-2018 – *Gas Transmission and Distribution Piping Systems* ([17])

    Art. 805.2.3, page 9,

    "*hoop stress, S_H [psi (MPa)]: the stress in a pipe of wall thickness, t [in. (mm)], acting circumferentially in a plane perpendicular to the longitudinal axis of the pipe, produced by the pressure, P [psig (kPa)], of the fluid in a pipe of diameter, D [in. (mm)], and is determined by* ***Barlow's formula*** *(U.S. Customary Units): $S_H = PD/2t$, where D is the nominal outside diameter of pipe (see 833.7, page 32) …*"

11. API Specification 5L – *Specification for Line Pipe* – 43rd edition – 2004 ([18])

    Appendix K.1, page 151, "*modified Barlow's equation in 9.4.3*", page 18

    "*As a measure to prevent distortion when testing at pressures equivalent to stresses in excess of 90% of specified minimum yield strength, the manufacturer may apply a calculation to compensate for the forces applied to the pipe end that produce a compressive longitudinal stress. The calculation in this appendix is* *based on Barlow's equation (see 9.4.3, $P = 2St/D$) modified by a factor based on the Maximum Shear Theory (see note). The calculation may be applied only when testing in excess of 90% of the specified minimum yield strength. In no case may the gage pressure for testing be less than that calculated using **Barlow's equation** at 90% of specified minimum yield strength.*

Note: The calculation is an approximation of the effective hoop stress (SE), which is practical for application under mill pipe testing conditions. Other calculations provide closer approximations of effective hoop stress but are complex and therefore impractical for application.

12. Troitsky, M.S. – *Tubular Steel Structures – Theory and Design* – Second Edition – The James F. Lincoln Arc Welding Foundation, 1990 ([40])

    Paragraph 9.3.1.2 at page 9-2:

    "*9.3.1.2 Barlow's Formula*

    An empirical formula for internal fluid pressure which gives results on the side of safety for all practical thickness ratios is that known as Barlow's formula. This formula is similar to the common formula except that the outside diameter of the pipe is used instead of the inside. Barlow's formula is $f_c = pD/2t$.

    While Barlow's formula is widely used because of its convenience of solution, it was not generally considered to have any theoretical justification until formulae based on the maximum-energy-of-distortion theory showed that for a thin-walled pipe with no axial tension, Barlow's formula actually is theoretically correct. Since most commercially important pipes have a ratio of wall thickness to outside diameter less than 0.10, Barlow's formula for thin-walled pipes is of great significance. Comprehensive bursting tests on commercial steel pipe have demonstrated that the formula predicts the pressure at which the pipe will rupture with an accuracy well within the limits of uniformity of commercial pipe thickness. In general, failure occurred at a pressure about three percent higher than predicted. Barlow's formula has been employed in the ASA Standards for Wrought Iron and Wrought Steel Pipe and the ASA Code for Pressure Piping."

The same equation is assigned to Mariotte in the following publications, all from Italian authors:

1. Annaratone, D. - *Pressure Vessel Design* – Springer, 2007 ([19])

   3.1 "*General Design Criteria*" page 47:

   "*Before discussing the problem based on the above considerations, it may be useful to recall Mariotte's well-known method. Let us consider the semi cylinder of unitary length shown in Fig. 3.1. The pressure resultant along x is*

$$F = pD_i \quad (3.1)$$

   whereas it is obviously zero along y.

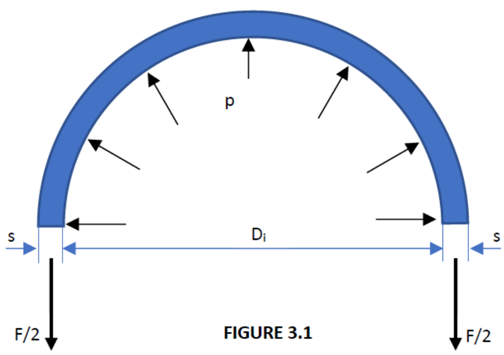


FIGURE 3.1

   We must apply two equal forces equal to F/2 at the ends of the semi cylinder to balance this thrust; if we assume that the hoop stress in the cylinder is constant through the thickness, we have:

$$\sigma_t = \frac{F/2}{s} = \frac{pD_i}{2s} \quad (3.2)$$

where s is the thickness.

If the hoop stress is equal to the basic allowable stress f we obtain

$$s = \frac{pDi}{2f} \quad (3.3)$$

Equation (3.3) is ***Mariotte's formula****, and it does not take into account the variation of σt through the thickness, as well as the presence of the other two principal stresses σ_r and σ_a; therefore it cannot be used for the sizing of the cylinder. …"*

In the frame of "*Cylinder Under Internal Pressure*", Page 60 (70),

From (3.84) we obtain the following equation, where D_m is the average diameter:

$$\frac{D_m}{s} = \frac{2f}{p} \quad (3.85)$$

and

$$s = \frac{pD_m}{2f} \quad (3.86)$$

Equation (3.86) is the ***so-called average diameter equation****; let us compare it with* ***Mariotte's*** *(3.3); with regard to the latter, the average diameter substitutes the inside one. Equation (3.86) can also be rewritten as follows: from (3.85), and considering the outside diameter*

$$p(D_e - s) = 2fs, \quad (3.87)$$

and hence

$$s = \frac{pD_e}{2f + p} \quad (3.88)$$

Equation (3.88) is used in many national codes and also in the ISO Code; a comparison with (3.81) ($s = pD_e/[2f + (1 + 0.15\,p/f)p]$) shows that they differ in the term between parentheses absent in (3.88).

Page 342 (350)

"*… In fact, (8.128) corresponds to the following:*

$$s_0 L' = \frac{p}{f}\frac{D_i}{2}L' \quad (8.129)$$

then

$$s_0 = \frac{pD_i}{2f} \quad (8.130)$$

also known as ***Mariotte's equation****.*

2. Vullo, V. – *Circular Cylinders and Pressure Vessels, Stress Analysis and Design* – Springer, 2014 ([20])

Mariotte mentioned in para. 1.2 page 5,

"*If, as is fairly frequent in design applications, external pressure is zero ($p_e = 0$) or internal pressure is zero ($p_i = 0$), Eq. (1.5) leads to the following respective relations:*

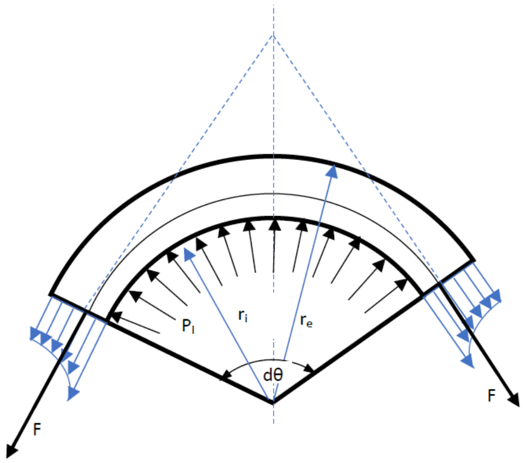
$$\sigma_t = \frac{p_i d_i}{2s} \text{ and } \sigma_t = \frac{p_e d_e}{2s} \quad (1.7)$$

These are ***Mariotte's formulas*** *for boilers.***"**

Para. 1.5 page 17

"*We will now consider radial equilibrium condition for the small element shown in Fig. 1.5, which is assumed to be of unitary axial length, under internal pressure p_i and open at the ends, so that we will have $\sigma_z = 0$. Following the same procedure used to arrive at* ***Mariotte's formulas*** *for boilers (1.7), but considering a circumferential small element of a circular cylinder having angular width $d\theta$ rather than π as in Fig. 1.1b and designating the resultant of stresses rt distributed along the radius as*

$$F = \int_{r_i}^{r_e} \sigma_t \, dr \ (1.22)$$



we obtain the relation

$$p_i r_i d\theta = 2F sin\,\frac{d\theta}{2} \quad (1.23)$$

which, given $sin\,(d\theta/2) \cong d\theta/2$, leads to:

$$p_i r_i = F \quad (1.24)$$

3. Risitano, A. – *Mechanical Design* – CRC Press, 201 ([21])

Page 331

"***Mariotte's generic formula*** *tells us that the tangential stress generated in cylindrical tubes undergoing internal pressure is*

$$\sigma_t = p^* \cdot d^*/2s$$

Page 424, Marriotte formula (yes, with two r):

"*For narrow pipes, the tangential tension σ_t is considered constant over the whole tube thickness (Figure 17.1). So, the* ***Marriotte formula*** *can now be applied:*

$$\sigma_t = \frac{p d_i}{2s}$$

where d_i is the internal pipe diameter, s is thickness, and p is the internal fluid pressure."

Pag. 427 Marriotte formula (again with two r), the same at page 660

"***17.4 SOME OIL PIPE CONSIDERATIONS***

Fuel pipes are subject to checks defined by the API standards to guarantee reliability, given the consequences of any loss. Generally, these pipes are thin, so for a nominal diameter of DN = 500 mm, thickness would be 6.5 mm. So, here the ***Marriotte formula*** *can be applied.*"

Also, the following university course notes claim for the "Mariotte's formula" (see the URL below); in some case, these notes claim also for an improbable "Boyle-Mariotte's formula":

http://dma.ing.uniroma1.it/users/broggiato/cdm/roma/no/ecdm/dispense-2012-13/11-Serbatoi.pdf	Mariotte's formula
http://unica2.unica.it/rdeidda//studenti/Acq_Fog_A4/Cap_A09_Tubazioni.pdf	Mariotte's formula
https://it.wikipedia.org/wiki/Tubo_per_condotte	Mariotte's formula
http://host.uniroma3.it/docenti/volpi/Infrastrutture_idra_2/Tubazioni.pdf	Mariotte's formula
http://www.dimnp.unipi.it/forte-p/Materiale_didattico/PAC_LMVeicoli/Cilindri%20in%20pressione.pdf	Mariotte's formula
https://it.wikipedia.org/wiki/Serbatoio_cilindrico	Mariotte's formula
http://corsiadistanza.polito.it/on-line/CMM/pdf/U6_L2.pdf	Boyle-Mariotte's formula

http://www.dimnp.unipi.it/leonardo-bertini/Corsi/CAC/Materiale%20didattico/Lez3-Gusci_sottili_assialsimmetrici.pdf	Boyle-Mariotte's formula
http://www.dimnp.unipi.it/leonardo-bertini/Corsi/CMM/Materiale%20Didattico/2013-14/Lezioni%20su%20BPVC.pdf	Boyle-Mariotte's formula
https://www.designapproval.org/design/calcolo-degli-spessori/	Boyle-Mariotte's formula

The internet search with key-words "*Mariotte's equation*" (>>) shows that the Mariotte formula as claimed here above appears only on Italian sites; on abroad sites the Mariotte's name is associated, along with Boyle's, to the perfect gases or compressible gases law. The confusion is increased by Risitano's handbook (ref. [21]) where at pages 424, 427 and 660 the "Marriotte formula" (with double r) is mentioned; while at page 331 the "Mariotte's generic formula" is given.

In this lexical confusion, the most ingenious (and funny) solution is the one proposed by the site http://www.larapedia.com/fisica_glossario/formula_di_Mariotte.html where you can read that the English translation of "Mariotte's formula" is "*Barlow's equation*".

A similar result is got with a web research run with French keywords, namely "*La formule de Mariotte pour les tuyauteries*" (>>), or "*La formule de Mariotte pour l'épaisseur des tuyauteries*" (>>). The first research outcome is related only to the compressed gas law; the second outcome returns something related to the piping thickness calculation, but in no case linked to Mariotte. Link to Mariotte are found in the following three cases, which anyhow have nothing to do with the hoop stress formula:

1. In book «*Traité de la construction des ponts*», Livre troisième, by Emiland-Marie Gauthey (Leduc, 1843) (>>), Mariotte is mentioned at page 19 in relation to the experiments carried out to determine the thickness of piping, without presenting any formula for this determination;
2. In book « Architecture hydraulique, ou l'art de conduire, d'élever et de ménager les eaux … », Tome second, by M. Belidor, 1782, Mariotte is referenced several times, with reference to his experiences with piping, but never the formula attributed to him is provided;
3. In handbook « *Manuel de l'Ingénieur des ponts et chaussées* » by A. Debauve (Dunod, 1875), Mariotte is mentioned at page 187 only for the compressed gas law; the equation to compute the thickness of a concrete conduct ($E = D \cdot H/30$ with D = diameter and H = pressure in m) shown at page 183 is conceptually similar to the one under discussion, but the author does not attribute it to Mariotte, as well as it happens at page 197 where the author presents the classical formula $hd = 2R \cdot e$, once again without mentioning Mariotte.

Even the research with keywords "*calcul d'épaisseur de tuyauterie en pression*" (>>) does not return any connection to Mariotte, as well as the web research with keywords "*Théorie de dimensionnement d'épaisseur de tuyauterie en pression*" (>>).

Even in current French sites and books coeval with Mariotte, in conclusion, there is no memory of the so called "Mariotte's formula" to compute the piping thickness.

A further analysis of the books coeval to Mariotte, performed following the bibliography of the book "*La colonne: nouvelle histoire de la construction*" by Roberto Gargiani (>>), provide the following outcome:

1. « *Histoire de l'Académie Royale des Sciences* », Année MDCCII (1702), Paris 1743 :
   - *a)* *« Sur la résistance des solides* », pp. 102-118 (pdf bnF Gallica pp. 117-133, >>, pdf Google pp. 120-136, >>) ;
   - *b)* *« Sur la résistance des cylindres creux et solides »*, p. 120 (p. 135 digital Edition by BnF Gallica, >>, and p. 138 digital Edition by Google, >>). Note: in some bibliography, this article is attributed to A. Parent, whereas Parent is mentioned as being the author of a formula for the sizing of solid and hollow cylinders.
   - *c)* Varignon, Pierre, « *De la résistance des solides en général pour tout ce qu'on peut faire d'hypothèses touchant la force ou la ténacité des fibres des corps à rompre; et en particulier pour les hypothèses de Galilée et de Mariotte »*, *Mémoires de l'Académie Royale des Sciences*, pp. 66-94 (pp. 222-250 BnF Gallica, >>, and pp. 226.254 pdf Google, >>)
2. *« Histoire de l'Académie Royale des Sciences »*, Année MDCCVII (1707), Paris 1730 :
   - a) *« Sur la résistance des tuyaux cylindriques pleins d'eau »*, pp. 126-131 (pp. 136-141 pdf BnF Gallica, >>, e pp. 142-147 pdf Google, >>). Note: This article too is in some bibliography attributed to Parent, whereas

Parent is once again mentioned as the author of a formula to determine the thickness of piping subjected to the water weight.

b) A. Parent, « *Des résistances des tuyaux cylindriques pour des charges d'eau et des diamètres donnés* », *Mémoire des Mathématiques et de Physique*, pp. 105-111 (pp. 321-327 pdf Google, >>, pp. 315-321 pdf BnF Gallica, >>), where the author (critically) review the Mariotte's rule and makes reference to the book «*Divers ouvrages de mathématiques et de physiques*» of the Académie Royale de Sciences, 1693 which published a paper from Mariotte «*Règles pour les jets d'eau*» pp. 508-509 (pp. 526-527 pdf BnF Gallica, >>)

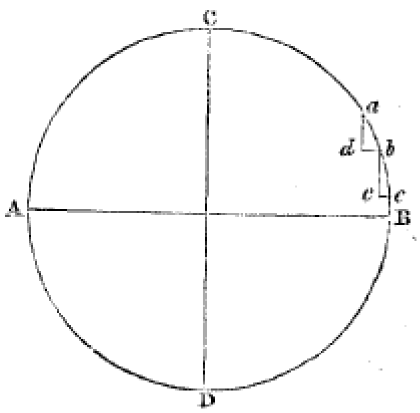
3. *« Histoire de l'Académie Royale des Sciences »*, Année MDCCIX (1709), Paris 1733 :
   a) Anonymous review, « *Sur un problème de statique* », pp. 109-112 (pp. 119-122 pdf BnF Gallica, >>, and pp. 129-132 pdf Google, >>) (Fonte : https://architettura.unige.it/bma/IT/AUTORI/it_autori_Varignon_P.html)
   b) Varignon, Pierre, « *Problème de statique* », *Mémoires de l'Académie Royale des Sciences*, Année 1709, Paris 1733, pp. 351-354 (pp. 502-505 pdf BnF Gallica, >>, and pp. 521-524 pdf Google, >>)
4. *« Histoire de l'Académie Royale des Sciences depuis son établissement en 1666 jusqu'à 1686 »*, Tome I
   a) E. Mariotte, « *Mathématiques, Hydrostatique* », pp. 69-72 (pp. 87-90 pdf BnF Gallica, >>)
   b) E. Mariotte, « *Hydrostatique* », pp. 170--172 (pp. 190-192 pdf BnF Gallica, >>)
   c) E. Mariotte, « *Mathématique (Géométrie, Mécanique, ecc.) … 2. Observation sur la résistance des tuyaux de conduite d'eau* », 1666, p. 225 (p. 245 pdf BnF Gallica, >>)
5. E. Mariotte, « *II. Discours, De la force des Tuyaux de conduite, et de l'épaisseur qu'ils doivent avoir suivant leur matière et la hauteur des réservoirs* », *Traité du mouvement des eaux*, pp. 348-382 (pp. 363-395 pdf Google >>, Wikisource >>);
6. E. Mariotte, Œuvres, pp. 460-473 (p. 510-523 pdf Google >>, p. 508-521 pdf BnF Gallica, Tome 1 >>)

All these papers have many references to Mariotte, but never attributing to him the paternity of the formula for piping hoop stress or thickness determination to which Parent and Varignon look rather to work and study.

Finally, it is noted that also the book testo «*Mariotte, savant et philosophe (1684): analyse d'une renommée*» by Pierre Costabel, (Google >>), even describing in detail Mariotte's activity on piping resistance, nevertheless does not mention any formula that can be attributed to him (page 120).

Barlow's Formula derivation

Barlow's method



Let's consider a cylinder subjected to internal pressure p. Let's divide the circumference into arches of infinitesimal length c as shown in the figure aside (from ref. [31]). On each arc element the pressure p is applied along the radial direction (normal to the arc) developing the radial force $F = p \cdot c$. This force can be resolved into a component parallel to the axis AB and another component parallel to the axis CD. Let's consider now the semicircle DBC. The sum of all forces parallel to AB shall be equal to the sum of all forces normal to the segments ad, bc and so on. The sum of the length of this segments is equal to the diameter CD. The sum of the force components parallel to AB is therefore equal to $p \cdot D$, where D is the diameter CD. This force is sustained by the thickness of material in C and D, from which it follows that the stresses (that Barlow designates as "*direct strains*" terms currently used for the unitary deformations, see ref. [6] and [31]) in D shall be equal to the product of the pressure by the radius (the cylinder is supposed to have unitary depth).

To compute the thickness required to sustain this force, Barlow adopted the following approach. Let us imagine the circular ring divided into a series of circumferences of very small thickness side by side each other. Let us consider all circumferences on the right of diameter CD. Each circular lamina, being subject to the stress developed by the pressure, experiences a stretch proportional to the stress. Since the external circular lamina contribute less to sustain the pressure, they are less stressed and less stretched. Because of the stretching the radii of each fiber increase, keeping the area of the section unchanged, since the quantity of material cannot change. Assuming that the internal diameter increases from D to $D + d$, and the external diameter increases from D' to $D' + d$, the constancy of the area gives:

$$D'^2 - D^2 = (D' + d')^2 - (D + d)^2$$

from which

$$D'^2 - D^2 = D'^2 + d'^2 + 2D'd' - D^2 - d^2 - 2Dd$$
$$d'^2 + 2D'd' = d^2 + 2Dd$$
$$d'(d' + 2D') = d(d + 2D)$$
$$(d' + 2D'):(d + 2D) = d:d'$$

Since d and d' are very smaller than D' and D, the above equation can be reduced to:

$$D':D = d:d'$$

Stretching d' of the external fiber is proportional to stretching d of the internal fiber as the inside diameter is proportional to the outside diameter. Setting the resistance as the ratio of the fiber stretch divided by its initial length, it follows that:

$$\frac{D'}{D} = \frac{d}{d'} \to \frac{d}{D} = \frac{d'D'}{D^2} \to \frac{d}{D} = \frac{d'}{D'} \cdot \frac{D'^2}{D^2}$$
$$\frac{d}{D}:\frac{d'}{D'} = D'^2:D^2$$

The fibers' resistance (to be read as deformation) decreases from inside to outside proportionally to the square of the ratio of the outside over inside diameter.

Said r the inside radius of the cylinder, p the pressure, t the metal thickness and x the radial coordinate measured from the internal surface, the unitary force on the internal surface is equal to $s = p \cdot r$ (this assumption is dimensionally correct only in case of $t = 1$). The unitary force is proportional to the resistance ($s \propto d/D$), defined as the ratio of stretch over the initial length, then applying the above conclusions to the radial positions r and $r + x$:

$$s:s_x = (r + x)^2:r^2$$

from which:

$$s_x = s \cdot \frac{r^2}{(r + x)^2}$$

The sum of all unitary forces acting through the thickness therefore is:

$$S = \int_0^t s \cdot \frac{r^2}{(r + x)^2} dx = s \cdot r^2 \left[\frac{-1}{(2-1)(r + t)^{2-1}} - \frac{-1}{(2-1)(r)^{2-1}}\right] = s \cdot r^2 \left[\frac{1}{r} - \frac{1}{(r + t)}\right] = \frac{srt}{r + t}$$

The sum of the unitary forces is then equal to the one developed by the unitary force s when uniformly distributed over the equivalent thickness $rt/(r + t)$.

If c is the metal resistance to cohesion and t_{req} is the required thickness for a cylinder subject to a pressure p, the above equations show that the material reaction to the end pressure force is equal to the product of c and the equivalent thickness over which the unitary force $s = pr$ is applied; in other words:

$$pr = c\frac{rt_{req}}{r + t_{req}}$$
$$pr + pt_{req} = ct_{req}$$
$$t_{req} = \frac{pr}{c - p}$$

This demonstration is based on the error to assume that the stress on the inside radius is equal to pr which is an assumption not satisfying the equilibrium.

Goodman 's demonstration

The demonstration that Goodman presents in his book (ref. [30]) is the following.

The radii of a cylinder subjected to internal pressure p increase their length because of the stretching to which they are submitted. Said n_x the unitary increase of length (the deformation or strain) at the radial position x, the internal and external radii change their initial values r_i and r_o, respectively, to the final values $r_i + n_i r_i = r_i(1 + n_i)$ and $r_o + n_o r_o = r_o(1 + n_o)$. Since the area of the section does not change, and assuming that with small deformation, the circular shape does not change too (which is the Barlow's hypothesis), we can write:

$$\pi({r_o}^2 - {r_i}^2) = \pi[{r_o}^2(1 + n_o)^2 - {r_i}^2(1 + n_i)^2]$$

from which it follows:

$${r_i}^2({n_i}^2 + 2n_i) = {r_o}^2({n_o}^2 + 2n_o)$$

Since n_x is very small, its square is negligible, so that:

$$\frac{{r_i}^2}{{r_o}^2} = \frac{n_o}{n_i}$$

The same conclusion of Barlow is obtained, that the deformations are proportional to the square of the reciprocal of their radii.

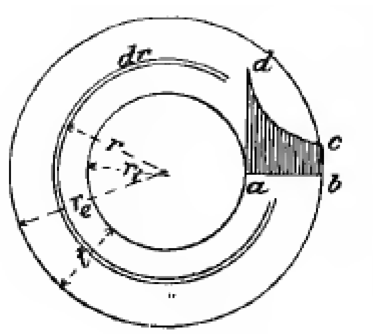


FIG. 404.

Since the material is elastic, the deformations are proportional to the stresses; therefore:

$$\frac{{r_i}^2}{{r_o}^2} = \frac{f_o}{f_i} \rightarrow f_i {r_i}^2 = f_o {r_o}^2$$

This equation allows the calculation of the stress acting on the circular lamina of infinitesimal thickness dr at the generic radius r:

$$f = \frac{f_i {r_i}^2}{r^2} dr$$

The sum of all stress acting through the entire thickness is:

$$S = \int_{r_i}^{r_o} \frac{f_i {r_i}^2}{r^2} dr = f_i {r_i}^2 \left[\frac{1}{r_i} - \frac{1}{r_o}\right] = f_i r_i - f_i \frac{{r_i}^2}{r_o} = f_i r_i - f_o r_o$$

Since this global stress is due to the pressure acting on the inside radius which generates the force pr_i, it follows:

$$pr_i = f_i r_i - f_o r_o = f_i r_i - f_i \frac{{r_i}^2}{r_o}$$

$$pr_o = f_i r_o - f_i r_i = f_i t$$

We obtain then a formula different than Barlow's one which did not properly account for the equilibrium. This formula is like the one for thin cylinders, with the difference that the internal radius is replaced by the outside radius and the stress at the internal radius replace the membrane (average) stress. Goodman's conclusion clarifies how the outside formula generated showing that it is based on inappropriate similitude since the stress handled in the thin cylinders is not the maximum value at the internal side, but the average one.

If in the above equation the internal radius of the second term is replaced the following is obtained:

$$pr_i = f_i r_i - f_o r_o = f_o \frac{{r_o}^2}{r_i} - f_o r_o = f_o \frac{{r_o}^2 - r_o r_i}{r_i} = f_o \frac{r_o}{r_i} t = \overline{f} t$$

We get again the same equation as for thin cylinder, with the internal radius and the average stress $\overline{f}$ (as for thin cylinder), as we should have expected since the force to be equilibrated in both cases act over the internal diameter.

Although resolving the issue of the missing equilibrium in the original Barlow's calculation, Goodman has not realized that, to get an effective equivalence between the two cases of thin and thick cylinders, it is necessary to consider the average stress (membrane) through the thickness. For thick cylinders, the average stress is:

$$\overline{f} = \frac{S}{t} = \frac{f_i r_i - f_o r_o}{t} = \frac{f_o \frac{{r_o}^2}{r_i} - f_o r_o}{t} = \frac{f_o \frac{{r_o}^2 - r_o r_i}{r_i}}{t} = f_o \frac{r_o}{r_i}$$

The effective comparison of the two cases should have been carried out in this way.

Discussion

The paragraphs above clearly show that the Barlow's and Mariotte's formulas, as used in literature, are practically identical.

The review of the original work of both scientists shows that Mariotte enunciated the rule without specifying neither the principle of the rule nor any formula, whereas Barlow obtained a formula that is quite similar to the one currently used by the design codes, but based upon a not correct assumption.

The Barlow's formula is almost exclusively recalled with this name by the pressure piping handbooks only. The pressure vessel handbooks, even though making use of the equation mathematics, never designate it as Barlow's formula. Practically all textbooks acknowledge as Barlow's formula that with the outside diameter (see Kellogg, ref. [9], Peng&Peng, ref. [10], Anvil, ref. [11], Ellenberger, ref. [12], Nayyar, ref. [13] , Becht, ref. [15], Rao, ref. [16] and [34] , ASME B31.8, ref. [17] , API 5L, ref. [18], Troitsky, ref. [40]).

For some authors, the formula was developed for thin cylinders (see, Ellenberger, ref. [12], Nayyar, ref. [13]); for others, it was developed for thick cylinders (see, Adams, ref. [22], ISO 10400, ref. [27]).

Nayyar ([13]) states that using the outer diameter in this formula was established by advanced analyses. Troitsky ([40]) states that the formula originally had an empirical background with no theoretical demonstration. ASME B31.8 ([17]) for piping in plastic material (article 842.2.1) makes use of the mean diameter.

For some authors (see, Zhu and Leis, ref. [39], e Troitsky, ref. [40]) the bursting pressure computed with the Barlow's formula has great reliability such to justify its large use. Other authors (see, Adams, ref. [22]) states that the formula has a poor reliability, especially for thin wall piping.

A so simple formula with a such muddled history!

Regarding the muddling aspect, it is interesting to observe that, because of the Barlow wrong hypothesis, described above, the denominator of Barlow's original formula does not contain the material strength only (the allowable stress), but the strength deducted the applied pressure. It appears that, moving from a wrong assumption, for the calculation of the required thickness, Barlow proposed a formula having the same structure than the one currently used by the modern design codes, the so-called **Boardman's formula** ([8] with $Y = 1$ (v. ref. [9]):

$$S = \frac{pR_i}{t} + Yp$$

Let us consider the design formula (1) of ASME VIII-1. Article UG-27:

$$t = \frac{PR}{SE - 0.6P}$$

If we set $c = SE$, being R the inside radius, that adopted by ASME VIII-1 is the Boardman formula with $Y = 0.6$, that coincides with the Barlow's formula as soon as we set $Y = 1.0$.

The same structure is adopted by formula (3) of ASME B31.1 art. 104.1.2

$$t = t_m - A = \frac{PD_o}{2(SE + Py)}$$

If we set $y = 0.4$ (applicable to all steel below 900°F, not in creep regime), we obtain the formula (1) of Appendix 1 of ASME VIII-1

$$t = \frac{PR_o}{SE + 0.4P}$$

Even though present in the Barlow's original formula, the pressure P disappears from the denominator in all cases where the Barlow's formula is explicitly identified as such. That occurs in textbooks, but also in codes (see, standard ASME B31.8, article 805.2.3). What is currently mentioned as Barlow's formula is that obtained by Goodman in his book ref. [30], where it becomes (D = outside diameter):

$$t = \frac{PR}{SE} = \frac{PD}{2SE}$$

If this equation is resolved for pressure, we obtain:

$$P = \frac{tSE}{R} = \frac{2tSE}{D}$$

This is the formula usually adopted for the bursting pressure calculation.

About whether the Barlow's equation applies to thin or thick piping, it is noted that according to ISO TR 10400 (ref. [27]), paragraph 6.6.2.1, the Barlow's formula "*represents the thin wall approximation to the biaxial VME / Lamé failure pressure. … The formula is for thick wall hoop stress, with failure taken to occur when the ID stress reaches yield. … Moreover, the derivation is incorrect because it violates the equilibrium condition.*" To support this statement, the standard refers to Goodman's book (ref. [30]) where, at page 421, paragraph "*Thick Cylinders*" the "*Barlow's Theory*" is discussed but without claiming any objection about the equilibrium violation.

The author explains that "*a thick cylinder may be dealt with by the same form of expression as a thin cylinder, taking the pressure to act on the external instead of on the internal radius.*" This statement clarifies where the outside diameter Barlow's formula sourced. The approach that Goodman used to get the final relationship is different than that used by Barlow. Presumably, it follows Goodman work the habit to apply the outside diameter Barlow's formula to thick wall piping (i.e. with ratio $D/t \leq 20$), whereas for thin wall piping the inside diameter Barlow's formula holds.

There is no doubt that the original formula written by Barlow made use of the inside radius, as well as there is no doubt that with the equilibrium considerations developed by Goodman (ref. [30]) he obtained the current formula where, for thick wall piping, the outside diameter is used as do almost all codes and authors (see summary table of last paragraph). According to Goodman (ref. [30]), the Barlow's theory (not the Barlow's formula that is disclaimed and … reformulated) applies only to thick cylinders. This limitation however does not appear in Barlow's original paper (ref. [31]) and seems more a Goodman's extrapolation.

The Goodman's equation using the outside diameter complies with the hypothesis that the collapse occurs when the inside edge stress reaches the yielding limit. In other words, this formula controls and limits the inner edge stress, not the stress averaged through the thickness that is lower; in this sense the equation obtained by Goodman is conservative.

On the other side, it can't be neglected that to obtain such a formula Goodman (as others) did not consider that the complete equivalence with the thin wall piping case is obtained only using the stress averaged through the thickness, not that at the inner edge, even though this value is the highest. If the equivalence is corrected in this way, even for thick cylinders the formula makes use of the inner diameter, as it is necessary for not violating the equilibrium.

Inhomogeneous comparisons and misleading conclusion should have been avoided simply focusing on the primary stress (which equilibrates the external loads, in this case the pressure) and on the membrane stress (averaged through the thickness) concepts, considering that plastic collapse is governed by primary membrane stresses, as all pressure vessel design codes acknowledge when requiring the check of the primary membrane stress and neglecting the local stress which are necessary for compliance not for equilibrium.

If the Barlow's formula expression intends to identify the equation that compute the hoop stress averaged through the thickness that equilibrates the pressure, such a formula shall make use of the inside (not outside) diameter even in case of thick-walled cylinders similarly to the thin-walled ones, where the average hoop stress is , $\overline{\sigma} = \sigma_o(r_0/r_i) = \sigma_i(r_i/r_o)$. This is an obvious conclusion since the equilibrium condition (the pressure act on the inside diameter) does not change because of the thickness dimension and stress distribution through the thickness. The Lamé theory, indeed, has general validity, so that the equation for thin wall are a limit solution of the general equations.

How the Barlow's formula is used in design codes

Design formulas and their derivation

Let us consider some of the main design codes and the design equations they use.

1.	ASME VIII-1	UG-27, equation (1)	$t = \dfrac{PR}{SE - 0.6P}, P \leq 0.385SE$
2.	ASME VIII-1	Appendix I-1, equation (1)	$t = \dfrac{PR_o}{SE + 0.4P}$
3.	ASME VIII-1	Appendix I-2, equation (1)	$t = R\left(e^{P/SE} - 1\right) = R_o\left(1 - e^{-P/SE}\right)$

4.	ASME VIII-2	Art. 4.3.3.1, equation (4.3.1)	$t = \frac{D}{2}\left(e^{P/SE} - 1\right)$
5.	ASME VIII-3	Art. KD-221.1, equations (KD-221.1) e (KD-221.2)	$P_D = min\left[2.5856 \cdot S_y ; 1.0773 \cdot \left(S_y + S_u\right)\right] \cdot \left(Y^{0.268} - 1\right), Y \leq 2.85$ $P_D = min\left(\frac{S_y}{1.25}; \frac{S_y + S_u}{3}\right) \cdot \ln(Y), Y > 2.85$
6.	ASME B31.1	Art. 104.1.2	$t = t_m - A = \frac{PD_o}{2(SE + Py)}$
7.	ASME B31.3	Art. 304.1.2	$t = \frac{PD}{2(SEW + PY)} = \frac{P(d + 2c)}{2[SEW - P(1 - Y)]}$
8.	ASME B31.8	Art. 841.1.1	$P = \frac{2St}{D} FET$ $P = \frac{2St}{D - t} FET, when\ D/t < 30$
9.	EN 13445-3	Art. 7.4.2, equations (7.4.1) and (7.4.2)	$e = \frac{PD_i}{2fz - P} = \frac{PD_e}{2fz + P}$
10.	EN 13480-3	Art. 6.1, equations (6.1.1) e (6.1.2)	$e = \frac{p_c D_o}{2fz + p_c} = \frac{p_c D_i}{2fz - p_c}, per\ D_o/D_i \leq 1.7$
11.	EN 13480-3	Art. 6.1, equations (6.1.3) e (6.1.4)	$e = \frac{D_o}{2}\left(1 - \sqrt{\frac{fz - p_c}{fz + p_c}}\right) = \frac{D_i}{2}\left(\sqrt{\frac{fz + p_c}{fz - p_c}} - 1\right),$ $per\ D_o/D_i > 1.7$

It is noted that, for thin thickness cases, the adopted formula has the same structure as the original Barlow's formula. On the contrary, the code B31.8 makes use of the formula that Goodman obtained applying Barlow's theory.

The common feature of these formulas is that the denominator has not only the allowable stress (the material strength), but its combination with the acting pressure using the sign plus or minus depending on the diameter used, if outside or inside.

Farr and Jawad, in their handbook aimed at guiding on codes ASME VIII-1 and VIII-2 application (ref. [32]), state that ASME added the terms "$0.6P$" and "$0.4P$" to empirically take into account the hoop stress nonlinear distribution through the thickness in case of thick-walled cylinder (i.e. with $t > 0.1R_i$) and then make the adequate correction to the results provided by the classical formula for thin-walled cylinders. When recalling this formula, the two authors refer to the handbook "*Mechanics of Materials*" by Beer and Johnston issued on 1992 (see ref. [33] page 478) where this equation is obtained with the usual equilibrium conditions, as all authors do, being Goodman the first (ref. [30]). The handbook "*Companion Guide to the ASME BPVC*" (ref. [34]), in paragraph 21.4.2.4, provides a similar explanation about term "$0.6P$" genesis.

As explained by the Kellogg (ref. [9], the factor "0.6" was proposed in 1944 by Boardman (ref. [8]

The European norm replaces the correction terms "0.6P" and "0.4P" with "0.5P". The term "0.5P" is obtained with the approach that Somnath Chattopadhyay describes at page 63, paragraph 5.4 "*Approximate equations*" of his textbook ref. [35]. As soon as the hoop stress equation for thin thickness is written with reference to the mean radius, R_m, and then developed with respect to the inside radius, it takes the structure of that used by EN 13445-3. In detail:

$$\sigma_H = \frac{PR_m}{t} = \frac{P\left(R_i - \frac{t}{2}\right)}{t} = \frac{PR_i}{t} - \frac{P}{2} \rightarrow t = \frac{PR_i}{\sigma_H - 0.5P}$$

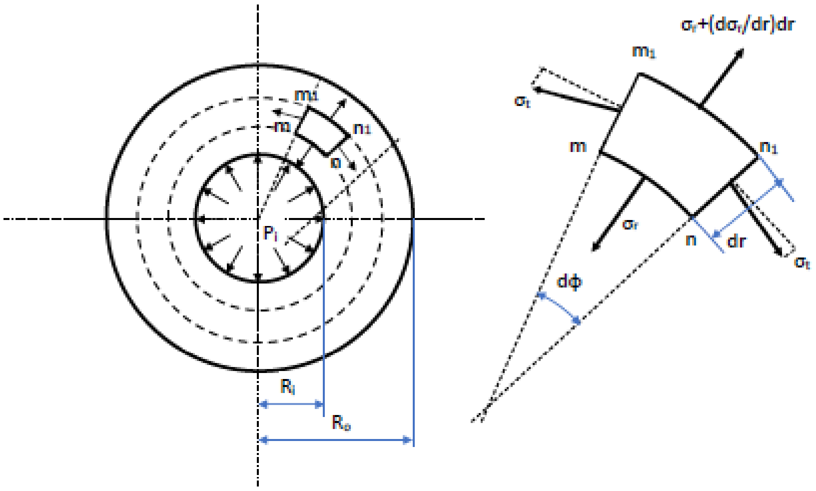
A similar explanation is provided by Bednar who in ref. [36], pages 46 and 47, obtains the equation here above and describes the source of the term "$0.6P$" adopted by ASME. The use of the mean radius in the initial equation is due to the membranal theory that the author uses to develop the equilibrium conditions. The correction that ASME adopted to be closer to the Lamé solution is therefore "$\pm 0.1P$" since "$0.5P$" is already considered by the membranal theory.

The membranal theory applied to thin shells assumes that the radial stress is negligible what, according to Bednar, is justified since on the average $\sigma_r = -P/2$ and with thin thickness P has usually small values.

On the other hand, the thin thickness is a limit case of the general solution (not thin thickness). In this last case, the assumption of negligible radial stress is no more valid, so that the equilibrium of the infinitesimal volume is obtained considering both the circumferential and the radial stresses, as Fryer and Harvey show in Figure 2.8.3 of their textbook (ref. [37], page 37). Upon writing the equilibrium equations and applying the boundary conditions, the Lamé solutions for the hoop and radial stresses are obtained. Said P the internal pressure, and R_i and R_o the inside and outside radii, the radial and hoop stresses are given by the formulas below:

$$\sigma_r = \frac{PR_i^2}{R_o^2 - R_i^2} - \frac{PR_i^2 R_o^2}{r^2(R_o^2 - R_i^2)}$$

$$\sigma_t = \frac{PR_i^2}{R_o^2 - R_i^2} + \frac{PR_i^2 R_o^2}{r^2(R_o^2 - R_i^2)}$$



It is meaningful to determine the mean values through the thickness obtained with these two equations:

$$\int \sigma_r dr = \int_{R_i}^{R_o} \frac{PR_i^2}{R_o^2 - R_i^2} dr - \int_{R_i}^{R_o} \frac{PR_i^2 R_o^2}{r^2(R_o^2 - R_i^2)} dr = \frac{PR_i^2}{R_o^2 - R_i^2}(R_o - R_i) - \frac{PR_i^2 R_o^2}{(R_o^2 - R_i^2)}\left(-\frac{1}{R_o} + \frac{1}{R_i}\right)$$

$$= \frac{PR_i^2}{R_o + R_i} - \frac{PR_i^2 R_o^2}{(R_o^2 - R_i^2)} \frac{R_o - R_i}{R_o R_i} = \frac{PR_i^2}{R_o + R_i} - \frac{PR_i R_o}{R_o + R_i} = \frac{PR_i^2}{R_o + R_i}\left(1 - \frac{R_o}{R_i}\right) = -\frac{PR_i t}{R_o + R_i}$$

$$\int \sigma_t dr = \int_{R_i}^{R_o} \frac{PR_i^2}{R_o^2 - R_i^2} dr + \int_{R_i}^{R_o} \frac{PR_i^2 R_o^2}{r^2(R_o^2 - R_i^2)} dr = \frac{PR_i^2}{R_o^2 - R_i^2}(R_o - R_i) + \frac{PR_i^2 R_o^2}{(R_o^2 - R_i^2)}\left(-\frac{1}{R_o} + \frac{1}{R_i}\right)$$

$$= \frac{PR_i^2}{R_o + R_i} + \frac{PR_i^2 R_o^2}{(R_o^2 - R_i^2)} \frac{R_o - R_i}{R_o R_i} = \frac{PR_i^2}{R_o + R_i} + \frac{PR_i R_o}{R_o + R_i} = \frac{PR_i^2}{R_o + R_i}\left(1 + \frac{R_o}{R_i}\right) = PR_i$$

The average stress is obtained diving by the thickness t the two above integrals, therefore:

$$\overline{\sigma_r} = -\frac{PR_i}{R_o + R_i}$$

$$\overline{\sigma_t} = \frac{PR_i}{t}$$

It is noted that the formula of the average hoop stress is the same obtained for the thin thickness case, as it must be since the average stress is obtained with equilibrium considerations.

Since there is no shear stress, both the above radial and hoop stresses are also main stresses. Using the Tresca criterion, the average equivalent stress is equal to their difference in sign; therefore:

$$\overline{\sigma}_{tr} = \frac{PR_i}{t} + \frac{PR_i}{R_o + R_i} = \frac{PR_i}{t} + \frac{P}{Y+1}$$

where $Y = R_o/R_i$

In case $R_o \cong R_i$, then $Y \cong 1.0$, and we get:

$$\overline{\sigma}_{tr} = \frac{PR_i}{t} + 0.5P \;\rightarrow t = \frac{PR_i}{\overline{\sigma}_{tr} - 0.5P}$$

which is exactly the equation that Bednar has obtained with the membranal theory and is adopted by EN 13445-3.

Applying the von Mises criterion, in lieu of Tresca, the membrane equivalent stress becomes:

$$\overline{\sigma}_{vm}{}^2 = \left(\frac{PR_i}{t}\right)^2 + \left(\frac{P}{Y+1}\right)^2 + \left(\frac{PR_i}{t}\right)\left(\frac{P}{Y+1}\right)$$

With $Y \cong 1.0$, we get:

$$\overline{\sigma}_{vm}{}^2 = \left(\frac{PR_i}{t}\right)^2 + \left(\frac{P}{1+1}\right)^2 + \left(\frac{PR_i}{t}\right)\left(\frac{P}{1+1}\right) = \left(\frac{PR_i}{t} + 0.5P\right)^2 - 0.5P^2\frac{R_i}{t}$$

$$= \left(\frac{PR_i}{t} + 0.5P\right)^2\left[1 - \frac{0.5P^2\frac{R_i}{t}}{\left(\frac{PR_i}{t} + 0.5P\right)^2}\right] = \left(\frac{PR_i}{t} + 0.5P\right)^2[1+X]$$

$$\overline{\sigma}_{vm} = \left(\frac{PR_i}{t} + 0.5P\right)\sqrt{1+X}$$

The X term may be written as follows:

$$X = -\frac{0.5P^2\frac{R_i}{t}}{\left(\frac{PR_i}{t} + 0.5P\right)^2} = -\frac{0.5\frac{R_i}{t}}{\left(\frac{R_i}{t} + 0.5\right)^2} = -\frac{0.5\frac{R_i}{t}}{\sqrt{0.5}\left(\frac{2R_i}{t} + 1\right)^2} = -\frac{\sqrt{0.5}\frac{R_i}{t}}{\left(\frac{2R_i}{t} + 1\right)^2} = -\frac{\sqrt{0.5}\frac{R_i}{t}}{1 + \frac{4R_i}{t} + \left(\frac{2R_i}{t}\right)^2 + \cdots}$$

Since $\lim_{R_i/t \to \infty} X = 0$, for the thin thickness case, also the von Mises criterion provides the same equation for the average equivalent stress:

$$\overline{\sigma}_{vm} = \frac{PR_i}{t} + 0.5P \;\rightarrow t = \frac{PR_i}{\overline{\sigma}_{vm} - 0.5P}$$

As the thickness increases, i.e. Y increases, in Lamé's equations, the radial stress coefficient, $\overline{\sigma}_r\,/\,P = -1/(Y+1)$, decreases in absolute value, but less quickly that the hoop stress coefficient, $\overline{\sigma}_t/P = 1/X = R_i/t$, so that the radial contribution to the membrane equivalent stress becomes progressively more important. In Fryer-Harvey textbook (ref. [37]), the Table 2.8.1 shows how the ratio of maximum (at inside radius) hoop stress to average hoop stress (obtained with thin piping equation) increases with the ratio $Y = R_o/R_i$. As Y moves from da 1.10 to 2.00, the ratio $\sigma_{t,max}/\sigma_{t,avg}$ moves from 1.05 to 1.67. Other interesting conclusions can be drawn if this approach is applied also to the radial stress and the Tresca's equivalent stress (indicated with σ_{eq}). The following table is the obtained:

$X = t/R_i$	0.010	0.020	0.040	0.050	0.060	0.080	0.100	0.200	0.400	0.500	0.600	0.800	1.000
$Y = R_o/R_i = (1+X)$	1.010	1.020	1.040	1.050	1.060	1.080	1.100	1.200	1.400	1.500	1.600	1.800	2.000
$Z = 1/(Y+1)$	0.498	0.495	0.490	0.488	0.485	0.481	0.476	0.455	0.417	0.400	0.385	0.357	0.333
$\overline{\sigma}_r\,/\,P = -Z$	-0.498	-0.495	-0.490	-0.488	-0.485	-0.481	-0.476	-0.455	-0.417	-0.400	-0.385	-0.357	-0.333
$\overline{\sigma}_t\,/\,P = 1\,/\,X$	100.000	50.000	25.000	20.000	16.667	12.500	10.000	5.000	2.500	2.000	1.667	1.250	1.000
$\overline{\sigma}_{eq}\,/\,P = 1\,/\,X - Z$	100.498	50.495	25.490	20.488	17.152	12.981	10.476	5.455	2.917	2.400	2.051	1.607	1.333
$\sigma_{r,i}\,/\,P = (1 - Y^2)\,/(Y^2 - 1)$	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000	-1.000
$\sigma_{t,i}\,/\,P = (1 + Y^2)\,/(Y^2 - 1)$	100.502	50.505	25.510	20.512	17.181	13.019	10.524	5.545	3.083	2.600	2.282	1.893	1.667
$\sigma_{eq,i}\,/\,P = \sigma_{t,i}\,/\,P + \sigma_{r,i}\,/\,P$	101.502	51.505	26.510	21.512	18.181	14.019	11.524	6.545	4.083	3.600	3.282	2.893	2.667
$\sigma_{t,1}\,/\,\overline{\sigma}_t$	1.005	1.010	1.020	1.026	1.031	1.042	1.052	1.109	1.233	1.300	1.369	1.514	1.667

$X = t/R_i$	0.010	0.020	0.040	0.050	0.060	0.080	0.100	0.200	0.400	0.500	0.600	0.800	1.000
$\sigma_{eq,i} \,/\, \overline{\sigma}_{eq}$	1.010	1.020	1.040	1.050	1.060	1.080	1.100	1.200	1.400	1.500	1.600	1.800	2.000
$\sigma_{r,o} \,/\, P \;=\; (1\;-\;1)\,/(Y^2\;-\;1)$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$\sigma_{t,o} \,/\, P \;=\; 2\,/(Y^2\;-\;1)$	99.502	49.505	24.510	19.512	16.181	12.019	9.524	4.545	2.083	1.600	1.282	0.893	0.667
$\sigma_{eq,o} \,/\, P = \sigma_{t,o} \,/\, P \;+ \sigma_{r,o} \,/\, P$	99.502	49.505	24.510	19.512	16.181	12.019	9.524	4.545	2.083	1.600	1.282	0.893	0.667
$\sigma_{eq,o} \,/\, \overline{\sigma}_{eq}$	0.990	0.980	0.962	0.952	0.943	0.926	0.909	0.833	0.714	0.667	0.625	0.556	0.500

Using von Mises' in lieu of Tresca's criterion, the equivalent stress becomes:

$$\sigma_{vm} = \sqrt{{\sigma_t}^2 + {\sigma_r}^2 - \sigma_t \sigma_r}$$

The previous table is then changed as follows:

$X = t/R_i$	0.010	0.020	0.040	0.050	0.060	0.080	0.100	0.200	0.400	0.500	0.600	0.800	1.000
$Y \;= R_o/R_i = (1 + X)$	1.010	1.020	1.040	1.050	1.060	1.080	1.100	1.200	1.400	1.500	1.600	1.800	2.000
$Z = 1/(Y + 1)$	0.498	0.495	0.490	0.488	0.485	0.481	0.476	0.455	0.417	0.400	0.385	0.357	0.333
$\overline{\sigma}_{vm}/P$	100.250	50.259	25.249	20.248	16.915	12.747	10.246	5.242	2.732	2.227	2.889	1.462	1.202
$\sigma_{vm,i}/P$	101.006	50.134	26.024	21.030	17.702	13.547	11.058	6.107	3.686	3.219	2.914	2.545	2.333
$\sigma_{vm,o}/P$	99.502	49.505	24.510	19.512	16.181	12.019	9.524	4.545	2.083	1.600	1.282	0.893	0.667
$\sigma_{vm,i}/\overline{\sigma}_{vm}$	1.008	0.998	1.031	1.039	1.047	1.063	1.079	1.165	1.349	1.445	1.543	1.741	1.941
$\sigma_{vm,o}/\overline{\sigma}_{vm}$	0.993	0.985	0.971	0.964	0.957	0.943	0.929	0.867	0.762	0.718	0.679	0.611	0.555

As expected, the difference between the edges' equivalent stress and the average equivalent stress is less pronounced, than Tresca case.

Based on what discussed, it is a bit weird the statement in paper ref. [22] that the Barlow's formula $P = 2ft/D_o$ is by chance (the authors use the term serendipity) obtained from the von Mises equivalent stress formula for plane states in case of thin wall.

On the contrary, it is, in my opinion, quite obvious that, when considering the membrane stresses, we get the Barlow's formula (as modified by Goodman) that is based on the equilibrium of membrane stresses. Not only this occurrence does not appear to be by chance, but it would be weird if not occurring.

What really surprising is that the original Barlow's formula (based on incorrect assumptions) has a structure strongly similar to that of the current design formulas, which are based on the membranal theory and on the Tresca's or von Mises' equivalent stress.

Bursting pressure

The Barlow's equation, as per Goodman modification, is largely used to compute the piping bursting or collapse pressure, even in presence of alternative equations able to provide more complete and precise solutions.

A search on the web with key-words "*piping burst pressure calculation*" gives the outcomes as per the link (>>). These links essentially develop the following considerations.

The equation for the bursting pressure calculation is in general given the following form:

$$P_b = \frac{2 \cdot S \cdot t}{(SF) \cdot D_o}$$

where:

- P_b is the pressure we are looking for, expressed in MPa or other units consistent with the adopted system,
- t is the piping thickness, in mm or another consistent unit,
- D_o is the piping outside diameter, in mm or another consistent unit,
- SF is the safety factor,
- S is the material strength, in MPa or another consistent unit.

The safety factor values depend upon the target of the calculation. When performing design calculations, SF is usually set equal to 1.5; when performing limit load calculations, $SF \;=\; 1.0$.

Even the material strength value depends upon the target of the calculation. When performing design calculations, S is usually equal to the allowable stress specified by the design standard (in this case the safety factor SF is still embodied in the allowable stress value); when performing limit load calculations, $S = S_Y$ (yielding strength) if the condition of plasticity onset is searched, or $S = S_U$ (ultimate tensile strength) if the limit condition is the bursting collapse. The first case is focused on the service limit state in elastic regime; the second case is focused on the collapse limit state.

The extensive use of the Barlow's equation to determine the bursting pressure is not, however, accepted by all the authors and norms, since, despite the undeniable advantages due to its simplicity, it has no general validity.

Beyond the search of a formula which is as precise and reliable as possible, under an engineering standpoint the following aspects are of great relevance:

1) the formula shall provide safe results under any service condition;
2) the formula shall be simple to minimize any possible error;
3) the results obtained shall be not excessively penalizing, to avoid becoming anti-economics (aspect highly sensitive for sectors at high density of capitals as *oil and gas*).

It is quite easy to realize that the Barlow's formula, if used with the yielding strength and the outside diameter, returns a collapse pressure value that certainly is a lower limit of those experimentally got (see, on this respect, Figure 3 in ref. [22] by A.J. Adams et al.).

Using this formula with the ultimate tensile strength can lead to an overestimated bursting pressure value. The overestimate increases as the material strain hardening exponent n increases. i.e. moving from ferritic to austenitic stainless steels.

This conclusion appears to be obvious, since:

1) The bursting tests looks for the collapse limit load, which is a condition inevitably obtained with a pressure value higher than that related to the elastic limit service load governed by the yielding strength.
2) Beyond yielding, being the behavior no more linear, the use of the ultimate tensile strength looks to be so much more inappropriate, how much the tensile curve is stretched and flattened. This happens for a simple physical reason: in materials with high strain hardening exponent the thickness has the tendency to reduce at a greater rate as the load increases, the strength capacity consequently reduces quicker and the critical stress is reached at pressures lower than those predicted by the Barlow's formula.

Therefore, the Barlow's formula is in general suitable for the calculation of the elastic service limit pressure, whereas it shall be used with attention for the calculation of the collapse service limit pressure, I.e. for the bursting pressure calculation.

Codes' requirements

The bursting pressure equation provided by the standard API TR 5C3 (ref. [23], [24]) is based on the Barlow's formula, as discussed by Halim et al. in [25] and Staelens et al. in [26]. For this application, the Barlow's formula makes use of the yielding strength and a safety factor of 0.875 (corresponding to the thickness tolerance of commercial piping). The bursting pressure so obtained is the elastic service limit state (plasticity onset condition) which represents the critical condition for threaded couplings of the so called OCTG (*Oil Country Tubular Goods*), where the fluid leakage may occur:

$$P_b = 0.875\frac{2S_Y t}{D_o}$$

Paragraph 6.6.2.2 of ISO TR 10400 (ref. [27]) makes use of the same formula from API to determine the elastic service limit pressure for thin-walled piping. In paragraph 6.6.4, the Barlow's formula is replaced by the criterion that the von Mises equivalent stress reaches the yielding at the inner edge.

In Appendix B.3.1 a modified Barlow's formula is given where use is made of the ultimate tensile strength (ULS) in lieu of yielding, with the advice that it applies only end capped piping:

$$P_b = p_{iR} = \frac{2f_u t}{D_o}$$

The equation that is considered to provide the best results is due to the work of Klever e Stewart, reff. [28] and [38]. It is based on the last one, with a corrective factor which considers the material strain hardening exponent n:

$$P_b = \frac{2f_u}{\frac{D_o}{0.875t} - 1}\left[\left(\frac{1}{2}\right)^{1+n} + \left(\frac{1}{\sqrt{3}}\right)^{1+n}\right]$$

Standard DNV-RP-F-101, aimed at assessing if corroded piping is fit for service, presents a formula where, in addition to the factor accounting for the flaw geometry Q, there is the following term:

$$P_b = P_{cap} = 1.05\frac{2f_u t}{D_o - t} \cdot F(Q)$$

In ref. [39], Zhu and Leis, while reviewing the various methods used to compute the bursting pressure, identify some of them, belonging to the so called Tresca family, which are substantially based on the Barlow's formula even though with some modification:

1. Modification 1	Based on tensile strength and outer diameter	$P_b = \frac{2\sigma_u t}{D_o}$
2. Modification 2	Based on tensile strength and internal diameter	$P_b = \frac{2\sigma_u t}{D_i}$
3. Modification 3	Based on flow stress $\sigma_{flow} = 0.5(\sigma_y + \sigma_u)$ and internal diameter	$P_b = \frac{(\sigma_y + \sigma_u)t}{D_i}$
4. Modification 4	DNV – Based on flow stress and internal diameter	$P_b = \frac{2\sigma_{flow} t}{D_m}$
5. Modification 5	From the authors based on tensile strength and mean diameter.	$P_b = \frac{2\sigma_u t}{D_m}$
6. Modification 6	Fletcher's equation based on flow stress, internal diameter, and the uniform strain at tensile strength	$P_b = \frac{2\sigma_{flow} t}{D_i(1 - \varepsilon_{UTS}/2)}$

It is noted that in addition to the six equations here above, Zhu and Leis list other fifteen equations, among which is one proposed by them. According to Zhu-Leis, modifications 1 and 2 provide reasonable enough predictions; however, modification 2 tends to overestimate about $+2.1\%$ the bursting pressure. Modifications 3 and 4 underestimate the bursting pressure in the range -6.8% and -9.2%. The industry usually makes use of the modification 1 since it provides conservative values for the bursting pressure, with an underestimation of about -3%. Modification 5 gives underestimates about -0.6%. This last result is however valid only nfor materials having ratio $Y/T = 0.7 \div 0.9$, where:

$$\frac{Y}{T} = \left(\frac{\varepsilon_{ys} e}{n}\right)^n$$

The values $Y/T = 0.7 \div 0.9$ represent carbon steels.

According to A. J. Adams et al. [22], this use of the Barlow's formula is not correct, since:

1) the formula was originally (1836) obtained for thick-walled piping using incorrect hypotheses (equilibrium violation);
2) it cannot be properly used for thin-walled piping for which unconservative results may be obtained. To overcome these issues, the ISO TC67 SC5 workgroup 2 has carried out the 2018 revision of the standard ISO TR 10400 where the Klever-Stewart's formula is used (ref. [28], [29]) since acknowledged to be the most accurate, as discussed by A. J. Adams et al. in [22].

Conclusions

The formula that Barlow originally (1836) obtained has a structure very similar to that used by the modern design codes that is based om Boardman's formula (ref. [8]).

The formula recalled as Barlow's formula was obtained by Goodman who applied the Barlow's theory to thick-walled cylinders and established that they are equivalent to thin-walled cylinders when the hoop stress values on the inner edge are equal. This equivalence criterion is the origin of the presence of the outside diameter in the formula commonly used.

In technical literature, there is no mention of a Mariotte's formula for the calculation of the wall thickness of piping subjected to pressure.

The structure of the design formulas used by the pressure vessel and piping codes is derived by the membranal theory applied to thin-walled shells which makes use of the mean diameter. However, the same formula may be derived for thick-walled piping if the membrane components of hoop and radial stresses are considered and the membrane Tresca equivalent stress is then obtained.

The Barlow's equation with the inside diameter is correct also for thick-walled piping if it is intended to represent the hoop stress averaged through the thickness (membrane stress), which is the stress that the design codes require to be controlled and limited to avoid the plastic collapse. It follows that no error is committed in using in this way the formula.

Depending on the diameter used, there are three different versions of the Barlow's formula:

1) With the internal diameter, the formula gives the primary membrane hoop stress; this formula applies to thin walled as well as to thick-walled cylinders.
2) With the outside diameter, the Barlow's formula gives the maximum hoop stress on the inner edge; this formula correctly applies only to thin walled cylinders since its precision decrease as the thickness increases.
3) With the mean diameter, the Barlow's formula assumes a structure very similar to that adopted by the modern design codes and is surprisingly like the original (1836) Barlow's formula, too.

The Barlow's formula, if used with the ultimate tensile strength and the mean diameter, is fit to determine the bursting pressure of piping made in ferritic steel, case for which it provides slightly underestimated values (-0.6%) what is safe. In case of steels with high values of the strain hardening exponent, n, as austenitic stainless steels are, it is necessary to use other more appropriate equations that consider this parameter (Zhu-Leis, Klever, Faupel, etc.).

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